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Transient Emittance Limit for Cooling a Semitransparent Radiating Layer

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Nomenclature

a	=	absorption coefficient of layer, m^{-1}
c	=	specific heat of radiating medium, W·s/kg·K
D	=	thickness of radiating layer, m
E_1, E_2, E_3	=	exponential integral functions
$F(X)$	=	shape of temperature distribution in product solution
n	=	refractive index of layer
q	=	heat flux, W/m^2
$\tilde{q}$	=	$q/\sigma T_{\rm ref}^4$
q_0	=	outgoing flux, W/m^2
T	=	absolute temperature, K
$T_{\rm ref}$	=	arbitrary reference temperature, K
T_m	=	integrated mean layer temperature, K
t	=	$T/T_{\rm ref}$
t_m	=	$T_m/T_{\rm ref}$
X	=	x/D
x	=	coordinate in direction across layer, m
$\varepsilon_{\rm fd}$	=	"fully developed" transient emittance of layer
$\varepsilon_{\rm ut}$	=	emittance of a layer at uniform temperature
κ_D	=	optical thickness of layer, aD
ρ	=	density of radiating medium, kg/m^3
ρ^i	=	internal reflectivity at a boundary
σ	=	Stefan–Boltzmann constant, W/m^2·K^4
τ	=	dimensionless time, $(4\sigma T_{\rm ref}^3/\rho c D)$(time)

Introduction

THIS note will show a special feature of transient radiative cooling of a plane layer in the limit of zero heat conduction. The refractive index of the layer is larger than one, and so the layer has internal reflections from its boundaries that are assumed diffuse. The layer is cooling in lower temperature surroundings, and so externally incident radiation is negligible. The transient emittance is the instantaneous radiative flux leaving one side of the layer divided by the blackbody flux corresponding to the layer instantaneous mean temperature. A limit is found where this emittance becomes constant for each optical thickness and refractive index of the layer. The emittance becomes constant, although the heat loss, mean temperature, and temperature distribution are all changing with time.

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Transient cooling of a plane layer with $n > 1$ in cold vacuum surroundings was calculated in Ref. 1 with an implicit finite difference procedure. After an initial transient period, the transient emittance became constant for each set of parameters in the limit of zero heat conduction in the layer. This had been shown in Refs. 2 and 3 for a plane layer and a two-dimensional rectangular solid with $n = 1$. This Note will show that a transient similarity solution exists when $n > 1$, which produces reflections within the layer. Fully developed emittance values $\varepsilon_{\rm fd}$ are evaluated from the similarity solution and are compared with values for a layer at uniform temperature. From the behavior of the transient solutions in Ref. 1 the $\varepsilon_{\rm fd}$ provide a lower emittance limit during transient cooling for the conditions considered here. The shape of the temperature distribution in the fully developed similarity region is found to be rather insensitive to both refractive index and optical thickness.

Analysis

A plane layer extending from $x = 0$ to $x = D$ is a gray-emitting, absorbing, and nonscattering medium with $n > 1$; heat conduction is small and is neglected. The layer is initially at uniform temperature and is placed in much cooler vacuum surroundings, and so energy is lost only by radiation. The surrounding temperature is low enough that radiation from the surroundings to the layer can be neglected.

The transient energy equation in dimensionless form is[1]

$$\frac{\partial t}{\partial \tau} = -n^2\kappa_D t^4(X,\tau) + \frac{\kappa_D}{2}\Big(\tilde{q}_0(\tau)\{E_2(\kappa_D X) + E_2[\kappa_D(1 - X)]\} + n^2\kappa_D\int_0^1 t^4(X^*,\tau)E_1(\kappa_D|X^* - X|)\,dX^*\Big) \tag{1}$$

Properties are assumed independent of temperature. The $\tilde{q}_0$ is the diffuse flux directed into the layer that is leaving either boundary by reflection. The $\tilde{q}_0$ was obtained in terms of t^4 in Ref. 4, where steady temperatures were calculated for a heated layer with $n > 1$:

$$\tilde{q}_0(\tau) = \frac{2n^2\rho^i\kappa_D\int_0^1 t^4(X,\tau)E_2(\kappa_D X)\,dX}{1 - 2\rho^i E_3(\kappa_D)} \tag{2}$$

A product solution is now tried: $t(X,\tau) = t(0,\tau)F(X)$, where $F(0) = 1$. The $F(X)$ is the shape of the transient temperature distribution; the success of the product solution shows that $F(X)$ remains fixed in the transient region being analyzed. The $t(1,\tau) - t(0,\tau) = t(0,\tau)[F(1) - 1]$ is the amplitude of the temperature distribution that changes with time as the layer cools. The product solution is substituted into Eq. (1) combined with Eq. (2) and the variables are separated to yield:

$$\frac{1}{t^4(0,\tau)}\frac{dt(0,\tau)}{d\tau} = -\frac{n^2\kappa_D}{F(X)}F^4(X) + \frac{\kappa_D}{2F(X)}\Big\{2n^2\rho^i\kappa_D\frac{E_2(\kappa_D X) + E_2[\kappa_D(1-X)]}{1 - 2\rho^i E_3(\kappa_D)} \times \int_0^1 F^4(X)E_2(\kappa_D X)\,dX + n^2\kappa_D\int_0^1 F^4(X^*)E_1(\kappa_D|X^* - X|)\,dX^*\Big\} \tag{3}$$

With the variables separated, each side of Eq. (3) must be a constant. A convenient expression for the constant is to use the right side of Eq. (3) at $X = 0$, noting that $F(0) = 1$. The constant is then equated to the right side of Eq. (3) and the result is an integral equation for $F(X)$:

$$F^4(X) = F(X)\left[1 - 2\rho^i \frac{1 + E_2(\kappa_D)}{1 - 2\rho^i E_3(\kappa_D)} \frac{\kappa_D}{2} \times \int_0^1 F^4(X)E_2(\kappa_D X)\,\mathrm{d}X - \frac{\kappa_D}{2}\int_0^1 F^4(X)E_1(\kappa_D X)\,\mathrm{d}X\right] + 2\rho^i \frac{E_2(\kappa_D X) + E_2[\kappa_D(1 - X)]}{1 - 2\rho^i E_3(\kappa_D)} \frac{\kappa_D}{2} \times \int_0^1 F^4(X)E_2(\kappa_D X)\,\mathrm{d}X + \frac{\kappa_D}{2} \times \int_0^1 F^4(X^*)E_1(\kappa_D|X^* - X|)\,\mathrm{d}X^* \tag{4}$$

The emittance is the energy flux leaving one side of the layer divided by σT_m^4. This gives $\varepsilon_{fd} = [(1 - \rho^i)/\rho^i]\bar{q}_0(\tau)/t_m^4(\tau)$. The $\bar{q}_0(\tau)$ is inserted from Eq. (2) and the product solution is substituted for $t_m(\tau)$. The result is that the emittance is independent of time and can be calculated from $F(X)$ and its integrated mean value $F_m = \int_0^1 F(X)\,\mathrm{d}X$:

$$\varepsilon_{fd} = \frac{n^2(1 - \rho^i)2\kappa_D}{1 - 2\rho^i E_3(\kappa_D)} \frac{\int_0^1 F^4(X)E_2(\kappa_D X)\,\mathrm{d}X}{F_m^4} \tag{5}$$

Comparisons of ε_{fd} can be made with the emittance ε_{ut} for a layer at uniform temperature found when F is constant in Eq. (5)

$$\varepsilon_{ut} = n^2(1 - \rho^i)\frac{1 - 2E_3(\kappa_D)}{1 - 2\rho^i E_3(\kappa_D)} \tag{6}$$

Equation (4) was solved for $F(X)$ by iteration.[2] To start, a parabolic shape was substituted for $F(X)$ on the right side, and evaluation using Gaussian integration yielded the next approximation for $F^4(X)$. No damping was required between iterations, and convergence was rapid in most cases. Convergence was somewhat slower for large κ_D, but required at most a few minutes on a Cray X-MP. Using 100 increments across the layer gave ε_{fd} values accurate to three significant figures.

Results and Discussion

Emittance values are in Table 1 and Fig. 1 for the "fully developed" transient condition (solid lines) and for a uniform temperature layer (dashed lines). The ε_{ut} values decrease with increasing n because of surface reflections; for $n = 4$ this effect is dominant and ε_{ut} is almost independent of κ_D. During

Table 1 Emittance values, $\varepsilon_{fd}(n, \kappa_D)$ and $[\varepsilon_{ut}(n, \kappa_D)]$

	κ_D				
n	1	2	5	10	20
1	0.772 (0.781)	0.894 (0.940)	0.777 (0.998)	0.551 (1.000)	0.336 (1.000)
1.5	0.811 (0.816)	0.866 (0.885)	0.811 (0.908)	0.678 (0.908)	0.499 (0.908)
2	0.790 (0.793)	0.818 (0.828)	0.790 (0.839)	0.713 (0.839)	0.589 (0.839)
2.5	0.750 (0.752)	0.767 (0.772)	0.750 (0.778)	0.704 (0.778)	0.619 (0.778)
3	0.707 (0.708)	0.717 (0.720)	0.707 (0.724)	0.677 (0.724)	0.621 (0.724)
3.5	0.665 (0.666)	0.671 (0.673)	0.665 (0.676)	0.645 (0.676)	0.607 (0.676)
4	0.626 (0.626)	0.631 (0.632)	0.626 (0.633)	0.613 (0.633)	0.586 (0.633)

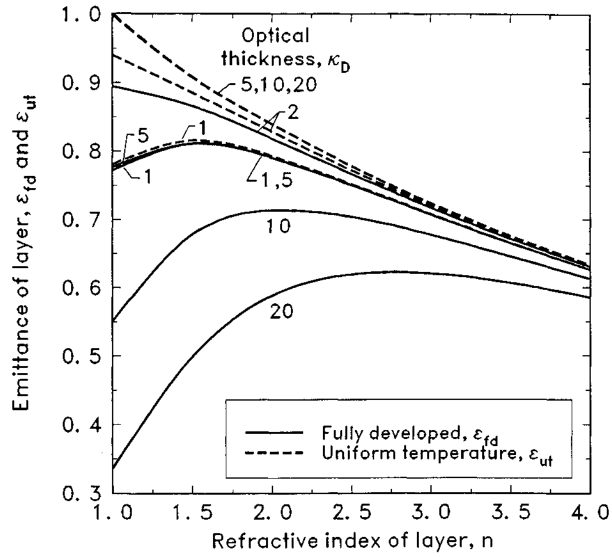
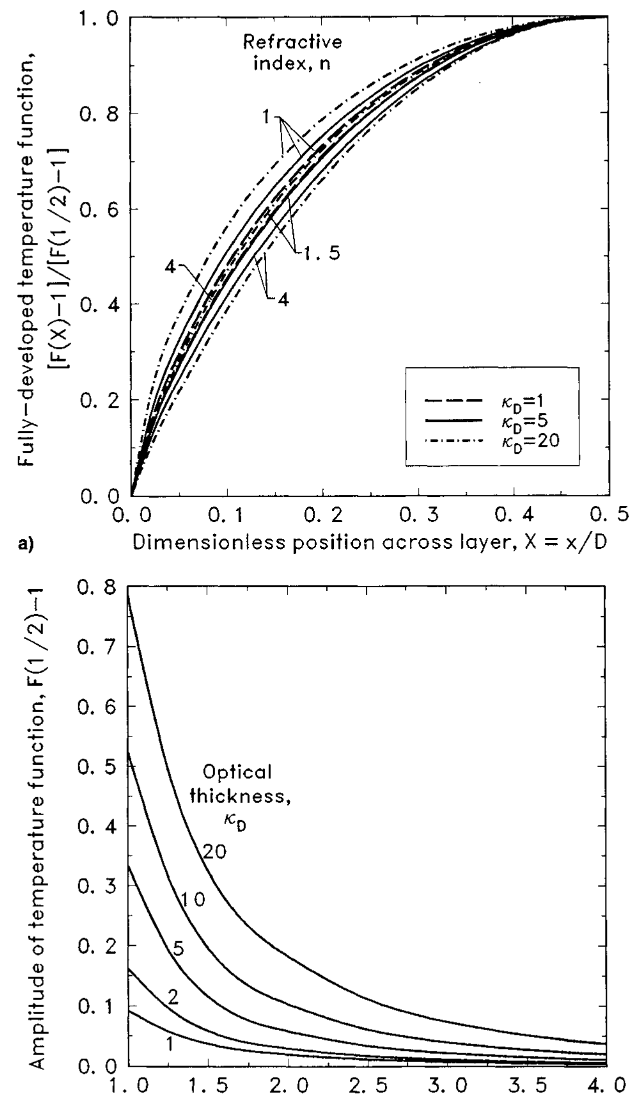


Fig. 1 Fully developed transient emittance as a function of refractive index and optical thickness compared with values for a uniform temperature layer.



a) b)

Fig. 2 Dimensionless temperature function for fully developed transient condition, as a function of refractive index and optical thickness: a) normalized shape of the temperature function and b) amplitude of the temperature function.

transient cooling the local temperatures near a boundary are lower than in the interior. The temperature variation is more pronounced for large κ_D and when n is near 1; this makes ε_{fd} much different from ε_{ut}. As n increases, internal reflections make the temperature distribution more uniform and ε_{fd} approaches ε_{ut}. For a fixed n, ε_{ut} increases with κ_D; for ε_{fd} however, there is a maximum for $\kappa_D \approx 2$.

The characteristics of the fully developed temperature distribution are in Fig. 2. The normalized shape of $F(X)$ is in Fig. 2a; it is rather insensitive to both n and κ_D. The effect of n on the shape is increased for larger κ_D values. The amplitude $F(1/2) - F(0) = F(1/2) - 1$ is in Fig. 2b. It increases substantially with κ_D if n is near 1. When $n = 4$, the amplitude is considerably reduced.

Conclusions

Fully developed transient emittances were obtained for a radiating layer cooling by exposure to a cold vacuum environment. The layer has diffuse surfaces and a refractive index $n \geq 1$. Emittances were evaluated from a similarity solution that was found to exist during transient cooling. As the layer temperature distribution decreases with time, its normalized shape becomes fixed for each set of parameters. The normalized shape is a weak function of n and κ_D. The fully developed transient emittances have a considerably different behavior than those for a uniform temperature layer. From the transient solutions in Ref. 1, the ε_{fd} provides a lower emittance limit during transient cooling for the present conditions.

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Spectrally Correlated Monte Carlo Formulations for Radiative Transfer in Multidimensional Systems

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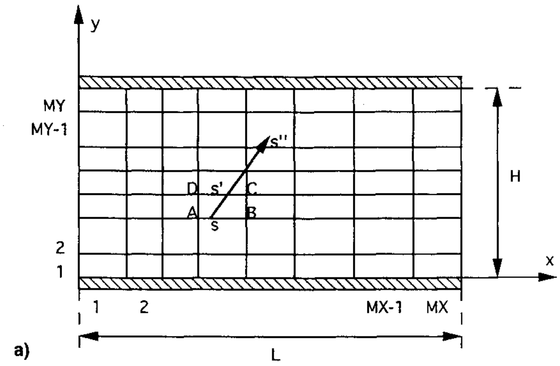
Introduction

FOR the investigation of radiative heat transfer in multidimensional gaseous systems, recent efforts are being directed to employ realistic narrow-band models to represent the absorption-emission characteristics of participating species. Use of a narrow band results in several types of spectral correlations in the radiative formulations. Three different methods[1-3] have been developed to deal with these spectral correlations. Among these methods, the Monte Carlo method (MCM) has been found to have definite advantages over other methods. In Ref. 3, radiative heat transfer between two infinite parallel plates was simulated in an exact manner, and one-dimensional correlated and noncorrelated Monte Carlo formulations were developed with almost no assumptions. However, the application of this exact treatment to multidimensional problems will be extremely complicated, and numerical solutions of these formulations will be very difficult. By introducing an appropriate assumption, the complicated Monte Carlo formulations can be simplified significantly. Therefore, the objective of this study is to develop and validate the approximate correlated and noncorrelated Monte Carlo formulations that are suitable for multidimensional problems. In this study, attention is directed to a simple two-dimensional problem. The exact Monte Carlo formulations are developed first and these are utilized to obtain the approximate formulations. Following this, a comparative study is conducted to determine the net radiative wall flux and radiative source distributions by employing the exact and approximate, correlated and noncorrelated, Monte Carlo formulations.

Analysis of Monte Carlo Simulation using a Narrow-Band Model

Consider an absorbing and emitting molecular gas between two parallel plates of finite length L and height H as shown in Fig. 1a. The inlet and outlet of the gas are at the section $x = 0$ and $x = L$, respectively, and they are treated as pseudoblack walls with prescribed temperatures. Temperature, concentration, and pressure in the medium are supposed to be known. The walls are assumed to be diffuse, but not necessarily gray. The temperature distribution of each wall is also known. The radiative transfer quantities of interest in this study are the net radiative wall flux and the radiative source term inside the medium. In order to calculate these quantities,



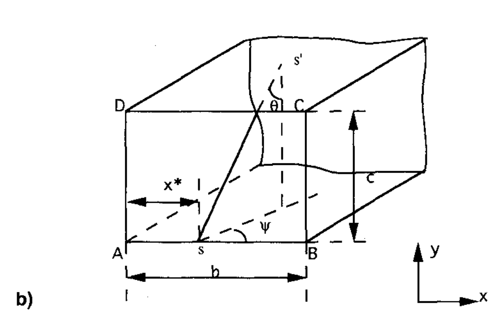


Fig. 1 Schematic of two finite parallel plates: a) coordinates system and b) volume element *ABCD*.

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